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Faraday Rotation and Depolarization of Galactic Radio Emission in the Magnetized Interstellar Medium

E. N. Vinyajkin

Radiophysical Research Institute (NIRFI),
 25 B. Pecherskaya st., Nizhny Novgorod, 603950, Russia

Abstract. A joint action of depth and bandwidth depolarization in the interstellar medium is considered using a model of N homogeneous synchrotron layers with Faraday rotation. The bandwidth depolarization can be used in multifrequency polarimetric observations of Galactic diffuse synchrotron radio emission to investigate the interstellar ionized medium and magnetic field in the direction to the Faraday-thick objects of known distances.

1 Introduction

Faraday rotation and depolarization have considerable impact on the angular pattern and frequency dependence of the position angle and brightness temperature of the linearly polarized component of the diffuse Galactic synchrotron radio emission. This effect increases with the distance from which we receive the linearly polarized radio emission. The observing frequency, bandwidth and beamwidth play important roles. Faraday depolarization may be caused by: 1) differential Faraday rotation along the line of sight when synchrotron emission and Faraday rotation are mixed (depth or front-back depolarization), 2) differential Faraday rotation in the receiver bandwidth (bandwidth depolarization), and 3) difference of Faraday rotation (and also intrinsic position angles) within the beamwidth (beamwidth depolarization). Depth and bandwidth depolarizations act at sufficiently low frequencies even in the case of a homogeneous radiation region and infinitely narrow antenna beam. Here we consider a joint action of depth and bandwidth depolarization in the interstellar medium using simple models of the regions with the synchrotron radio emission and Faraday rotation. We assume that 1) the receiver bandwidth $\Delta\nu \ll \nu_0$ (ν_0 is the central frequency), 2) the receiver frequency response is rectangular

$$\begin{aligned} \mathcal{F}(s) &= 1, \quad \text{if } |s| \leq \frac{\Delta\nu}{2\nu_0}, \\ \mathcal{F}(s) &= 0, \quad \text{if } |s| > \frac{\Delta\nu}{2\nu_0}, \end{aligned} \tag{1}$$

where $s = (\nu - \nu_0)/\nu_0$, 3) the beam is narrow enough to neglect the difference between position angles of waves coming from different directions.

2 Depth and bandwidth depolarization

2.1 A homogeneous region behind the Faraday screen

Let us consider a model consisting of a radio emission region of extension L along the line of sight with a homogeneous magnetic field and homogeneous space distributions of relativistic and thermal electrons and some other object located in front of it with substantial Faraday rotation and nonpolarized or negligibly small self-emission. Such an object can be an H II region, a magnetic bubble (Vallée, 1984), a planetary nebula, an external part of a molecular cloud (Uyaniker & Landecker, 2002; Wolleben & Reich, this volume), a depolarized supernova remnant (SNR), the solar corona (Soboleva & Timofeeva, 1983; Mancuso & Spangler, 2000), or the Earth ionosphere. The near object is the Faraday screen for

the region located behind it. Stokes parameters Q and U in this model account for depolarization in the rectangular bandwidth (1) (Vinyajkin & Krajinov, 1989)

$$\begin{aligned} Q &= \frac{P_0 I}{2\phi\lambda^2} \left(F [2(\phi + \phi_s)\lambda^2\delta] \sin\{2[\chi_0 + (\phi + \phi_s)\lambda^2]\} - \right. \\ &\quad \left. - F(2\phi_s\lambda^2\delta) \sin[2(\chi_0 + \phi_s\lambda^2)] \right), \\ U &= \frac{P_0 I}{2\phi\lambda^2} \left(-F [2(\phi + \phi_s)\lambda^2\delta] \cos\{2[\chi_0 + (\phi + \phi_s)\lambda^2]\} + \right. \\ &\quad \left. + F(2\phi_s\lambda^2\delta) \cos[2(\chi_0 + \phi_s\lambda^2)] \right), \end{aligned} \quad (2)$$

where P_0 is the intrinsic polarization degree, χ_0 is the intrinsic position angle, I is the intensity, $\phi = 0.81(\text{rad} \cdot \text{m}^{-2}\text{cm}^3\mu\text{G}^{-1}\text{pc}^{-1})N_e B_{\parallel} L$ is the Faraday depth of the radiation region, N_e is the electron density, $B_{\parallel} = \mathbf{B}\mathbf{k}/k$ is the component of the magnetic field \mathbf{B} along the line of sight (\mathbf{k} is the wave vector, $k = 2\pi/\lambda$ is the wave number), $\phi_s = 0.81(\text{rad} \cdot \text{m}^{-2}\text{cm}^3\mu\text{G}^{-1}\text{pc}^{-1})(N_e)_s B_{\parallel s} L_s$ is the Faraday screen depth, $\delta = \Delta\nu/\nu$, and

$$F(2x\lambda^2\delta) = \frac{\sin(2x\lambda^2\delta)}{2x\lambda^2\delta}. \quad (3)$$

2.2 N different homogeneous layers

Now let us consider a more general model consisting of N different homogeneous layers (Sokoloff et al., 1998), each of them characterized by three parameters: the intensity I_i , Faraday depth ϕ_i , and intrinsic position angle χ_{0i} , where $i = 1, 2, \dots, N$ and the farthest region has $i = 1$. If one of the layers is not a source of linearly polarized radio emission and only rotates the polarization plane, then $I_i = 0$. The intrinsic polarization degree of any emitting layer is the same and equals to P_0 .

Expressions for Stokes parameters of the N -layer model are easily obtained from (2) and (3), if we take into account that all regions from $i + 1$ up to N play the role of the Faraday screen for the i -region and rotate the polarization plane by the angle $\left(\sum_{j=i+1}^N \phi_j\right)\lambda^2$. Because the Stokes parameters are additive for noncoherent radio emission, the values of Q_N , U_N for the N -layer region can be obtained by summing up over all N components (Vinyajkin et al., 2002):

$$\begin{aligned} Q_N &= P_0 \sum_{i=1}^N \frac{I_i}{2\phi_i\lambda^2} \left(F \left[2 \left(\phi_i + \sum_{j=i+1}^N \phi_j \right) \lambda^2 \delta \right] \sin \left\{ 2 \left[\chi_{0i} + \left(\phi_i + \sum_{j=i+1}^N \phi_j \right) \lambda^2 \right] \right\} - \right. \\ &\quad \left. - F \left[2 \left(\sum_{j=i+1}^N \phi_j \right) \lambda^2 \delta \right] \sin \left\{ 2 \left[\chi_{0i} + \left(\sum_{j=i+1}^N \phi_j \right) \lambda^2 \right] \right\} \right), \\ U_N &= P_0 \sum_{i=1}^N \frac{I_i}{2\phi_i\lambda^2} \left(-F \left[2 \left(\phi_i + \sum_{j=i+1}^N \phi_j \right) \lambda^2 \delta \right] \cos \left\{ 2 \left[\chi_{0i} + \left(\phi_i + \sum_{j=i+1}^N \phi_j \right) \lambda^2 \right] \right\} + \right. \\ &\quad \left. + F \left[2 \left(\sum_{j=i+1}^N \phi_j \right) \lambda^2 \delta \right] \cos \left\{ 2 \left[\chi_{0i} + \left(\sum_{j=i+1}^N \phi_j \right) \lambda^2 \right] \right\} \right). \end{aligned} \quad (4)$$

If $\delta \rightarrow 0$, $F \rightarrow 1$, and Eqs. (4) correspond to eq. (9) from Sokoloff et al. (1998).

$N = 1$

In the case of a single homogeneous layer ($N = 1$) we get from (4):

$$\begin{aligned} Q_1 &= P_0 \frac{I}{2\phi\lambda^2} \left(\frac{\sin(2\phi\lambda^2\delta)}{2\phi\lambda^2\delta} \sin[2(\chi_0 + \phi\lambda^2)] - \sin 2\chi_0 \right), \\ U_1 &= P_0 \frac{I}{2\phi\lambda^2} \left(-\frac{\sin(2\phi\lambda^2\delta)}{2\phi\lambda^2\delta} \cos[2(\chi_0 + \phi\lambda^2)] + \cos 2\chi_0 \right). \end{aligned} \quad (5)$$

The depolarization factor $DP = \sqrt{Q^2 + U^2}/IP_0 = P/P_0$, where P is the observed degree of polarization, is equal to (Vinyajkin & Krajinov, 1989; Vinyajkin & Razin, 2002)

$$DP_1 = \frac{1}{2|\phi|\lambda^2} \left(\left[\frac{\sin(2\phi\lambda^2\delta)}{2\phi\lambda^2\delta} \right]^2 - 2 \frac{\sin(2\phi\lambda^2\delta)}{2\phi\lambda^2\delta} \cos(2\phi\lambda^2) + 1 \right)^{1/2}. \quad (6)$$

The dotted lines in Figs. 1 and 2 represent the dependencies of the depolarization factor (6) on $\phi\lambda^2/2$ in the intervals $0 \div 10$ and $70 \div 25\pi$, respectively, for the typical value $\delta = 0.01$. The solid lines in these figures show the dependencies of the observed position angle $\chi_{1\text{obs}}$ on $\phi\lambda^2/2$ ¹

$$\chi_{1\text{obs}} = \frac{1}{2} \text{angle}(Q_1, U_1), \quad (7)$$

where $\text{angle}(x, y)$ gives the angle in radians between the axis x (vertical axis Q) and the vector with coordinates x, y (polarization vector on the plane Q, U). It is seen from Eqs. (5) and (6) and from Figs. 1 and 2 that the oscillation amplitude of the position angle near the value $(\chi_0 + \pi/2)/2 = \pi/4$ (assuming $\chi_0 = 0$) decreases with increasing $\phi\lambda^2/2$, and at $\phi\lambda^2/2 = \pi/4\delta = 25\pi$ the position angle $\chi_{1\text{obs}} = \pi/4$, $DP_1 = \delta/\pi = (1/\pi)\%$.

In the limit of infinitely narrow band $\delta \ll 1/2|\phi|\lambda^2$ (we have assumed $\delta \ll 1$) Eqs. (5) transform to

$$\begin{aligned} Q_1(\delta \rightarrow 0) &= P_0 I \frac{\sin \phi\lambda^2}{\phi\lambda^2} \cos \left[2 \left(\chi_0 + \frac{\phi}{2} \lambda^2 \right) \right], \\ U_1(\delta \rightarrow 0) &= P_0 I \frac{\sin \phi\lambda^2}{\phi\lambda^2} \sin \left[2 \left(\chi_0 + \frac{\phi}{2} \lambda^2 \right) \right], \end{aligned} \quad (8)$$

and (6) transforms into the known formula for the depolarization factor of a homogeneous synchrotron layer with rotation in the limit of the infinitely narrow bandwidth and beam (Razin, 1956)

$$DP_1(\delta \rightarrow 0) = \left| \frac{\sin \phi\lambda^2}{\phi\lambda^2} \right|. \quad (9)$$

The position angle corresponding to Stokes parameters (8) equals to (Razin, 1956; Burn, 1966; Vinyajkin, 1995)

$$\chi_1(\delta \rightarrow 0) = \chi_0 + \frac{\phi}{2} \lambda^2 - \frac{\pi}{2} E(\phi\lambda^2/\pi), \quad (10)$$

where $E(x) = -E(-x)$ is the integral part of argument x . The position angle values of (10) may come out of the interval $0 \div 180^\circ$, for example, if $\chi_0 > \pi/2$ and $\phi > 0$. To calculate the observed values $\chi_{1\text{obs}}$ one has to use Eq. (7) with the Stokes parameters from (8). Figures 3 and 4 give plots of $\chi_{1\text{obs}}(\delta \rightarrow 0)$ as dependent on $(\phi/2)\lambda^2$ (solid lines) for the values of χ_0 , respectively, $\pi/4$ and $3\pi/4$ (dashed

¹ Here the observed values of the position angle are those measured in the interval $0 \div \pi$ and counted counter-clockwise from the vertical. In Figs. 1 and 2 the observed position angles are identical with the true ones. In the general case the observed value of the position angle may differ from the true one by $\pm n\pi$ ($n = 0, 1, 2, \dots$).

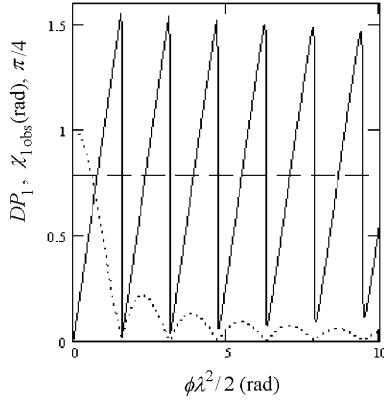


Fig. 1. The depolarization factor DP_1 (dotted line), the observed position angle $\chi_{1\text{ obs}}$ (solid line), and the number $\pi/4$ (dashed line), see text.

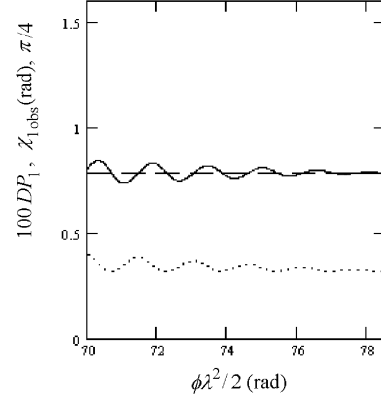
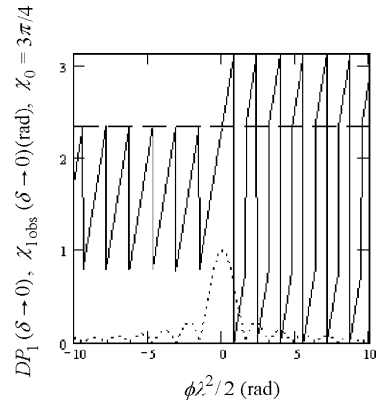
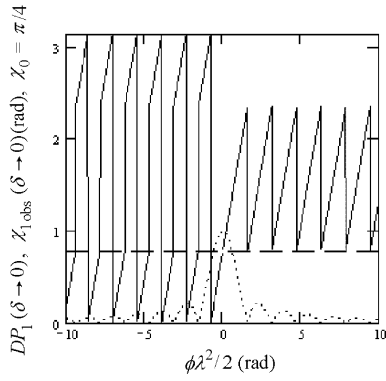


Fig. 2. Same as Fig.1, but for the $\phi\lambda^2/2$ interval 70 to 25π instead of 0 to 10, and $100 DP_1$ instead of DP_1 .



lines). The depolarization factor (9) is shown by dotted lines. The rotation measure $RM = \phi/2$ and peculiarities of its determination in this model have been considered in detail by Vinyajkin (1995). The model of a homogeneous layer was used by Vinyajkin (1995) to give the interpretation of a deep minimum of the polarization brightness temperature and a $\pi/2$ -jump of the position angle observed in the North Polar Spur in the direction with coordinates $\alpha_{1950} = 16^{\text{h}} 48^{\text{m}}$, $\delta_{1950} = 14^{\circ}$ at 960 MHz (Vinyajkin, 1995).

$N = 3$

Let us consider the model of the region consisting of two homogeneous synchrotron polarized layers and the Faraday screen between them. In this case the contribution of the far layer ($i = 1$) in the observed polarized radio emission becomes 0 at some relatively low frequency because of its bandwidth depolarization. If the distance to the Faraday screen is known, we can estimate the extension of the near synchrotron layer. As an example, let us consider the following model parameters: $I_1/I = I_3/I = 0.5$, $I_2 = 0$, $\chi_{01} = \chi_{03} = 0$, $\phi_1 = \phi_3 = 0$, $\phi_2 = 100 \text{ rad/m}^2$. The contribution of the far layer becomes 0 and, hence, the depolarization factor becomes 0.5 (see Figs. 5 and 6) at the minimum wavelength

$$\lambda_{\min} = \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{\phi_2 \delta}}, \quad (11)$$

which corresponds to the first zero of the function $|\sin(\Delta\chi)/\Delta\chi|$, where $\Delta\chi = 2\phi_2\lambda^2\delta$ is the differential Faraday rotation in the bandwidth. Substituting $\phi_2 = 100 \text{ rad/m}^2$, $\delta = 0.01$ in (11) we get $\lambda_{\min} \approx 1.25 \text{ m}$ ($\nu_{\max} \approx 240 \text{ MHz}$). Equation (11) can be used to estimate the cut-off wavelength of the far layer if $\phi_1 \ll \phi_2$. Let us consider some objects. The RM of SNR CTB 104A changes from $\sim -80 \text{ rad/m}^2$ in the southeast to $\sim +170 \text{ rad/m}^2$ in the northwest (Uyaniker et al., 2002). Assuming $\phi_2 \sim 340 \text{ rad/m}^2$, $\delta = 0.01$ we get from (11) $\lambda_{\min} \sim 0.7 \text{ m}$ ($\nu_{\max} \sim 430 \text{ MHz}$). At this wavelength this part of the SNR is nearly completely depolarized because of the depth depolarization ($P < 0.4\%$). The RM of the H II region S205 is 250 rad/m^2 (Mitra et al. 2003; Wielebinski & Mitra, this volume). In this case $\phi_2 = 250 \text{ rad/m}^2$ and, if $\delta = 0.01$, we get $\lambda_{\min} \approx 0.8 \text{ m}$ ($\nu_{\max} \approx 375 \text{ MHz}$). Gray et al. (1999) detected a strong beam and bandwidth depolarization across the face of W3 and W4 and immediately near them at 1420 MHz (30 MHz bandwidth).

Carrying out high angular resolution broadband polarimetric multifrequency observations in the directions to the Faraday screens with known distances it is possible to investigate the interstellar ionized gas and magnetic field along the line of sight. However, interference is a serious problem in carrying out such observations.

3 Conclusion

The bandwidth depolarization can be a useful tool in multifrequency polarimetric observations of Galactic diffuse synchrotron radio emission to investigate the interstellar ionized gas and magnetic field in the directions to Faraday-thick objects of known distances.

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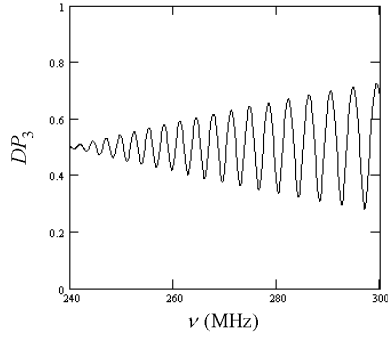


Fig. 5. The depolarization factor DP_3 of the three-layer model (see text) versus frequency in the interval $240 \div 300$ MHz.

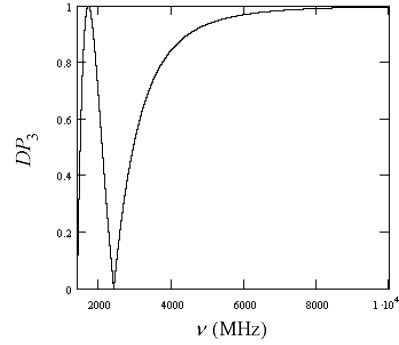


Fig. 6. Same as Fig. 5, but for the frequency interval $1400 \div 10000$ MHz.

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